# Joint Network and Dirty-Paper Coding for Multi-way Relay Networks with Pairwise Information Exchange

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***Abstract*—In this paper two-stage decode-and-forward (DF) coding schemes for relay networks with multi-pair information exchange . A basic coding element called joint network and dirty-paper coding (JNDPC) method is proposed for the broadcast stage. The JNDPC method embeds network coding into dirty-paper coding, which allows interference cancelation while fully utilizing the side information of each user. By using JNDPC, we propose a novel successive network coding (SNC) scheme that consists of two layers. In the first layer, JNDPC is used to cancel interference at the encoder. In the second layer, JNDPC is employed successively to facilitate successive decoding at the decoders. In the SNC scheme, the interference to each pair of users are canceled simultaneously, thus achieving larger rate region than joint network and superposition coding scheme. With JNDPC method, we then improve the performance of joint physical-network coding scheme which, together with joint network and superposition coding schemes, constitute the state of the art coding schemes.For the multiple access stage, a full decode multiple access scheme and a functional decode multiple access scheme are presented. The achievable regions of all proposed BC and MA stage coding schemes are established.**

## I. Introduction

Multi-way relay networks, where users exchange packets with the help of a relay, are regarded as a means of cooperating communications. Such networks naturally appear in satellite communications and provide robustness for wireless network systems. In the multi-way relay networks, each user knows a priori the packet it sends as side information. Such side information can be leveraged to increase the packet transmission rate. Various transmission protocols have been proposed for the two-way relay [1]–[5], which is the two user case of the multi-way relay. The protocols, including amplifying -and-forward schemes (AF) [1], decode-and-forward schemes (DF) [1]–[5], and compress-and-forward schemes (CF) [2], [3], generally consist of an multiple access (MA) stage and a broadcast (BC) stage. In the MA stage, the users transmit signals to the relay. In the BC stage, the relay broadcasts to the users. Lattice coding strategies are proposed in [4], [5] and shown to achieve the symmetric rate within half a bit.

Generalization of the two-way relay to the multi-user cases are also investigated. The work in [6] studied the multi-way relay networks with cluster data exchange, where there are $L$ clusters, each having $K$ users exchange data with each other. Relaying with pairwise packet exchange, which is the $K = 2$ case of the cluster data exchange, is considered in [7], [8]. Usually joint network coding and superposition coding (JNSC) [9], joint physical-network coding (JPNC) [10], and time sharing among these schemes are used in the BC stage for relaying with pairwise exchange. In the JNSC scheme, the network coded packets of each pair are superimposed for broadcasting and successive decoding is employed at the users to cancel interference. The interference cancelation efficiency at each user is constrained by its noise power. In the JPNC scheme, a separated network and dirty-paper coding method is employed and self information (side information) canceling is further used for interference cancelation. The achievable rate region of this scheme is also limited since the network and channel coding are separated.

In this paper we consider relay networks with pairwise packet exchange, where two-stage DF schemes are assumed. In contrast to the separate network and dirty-paper coding in JPNC scheme, we propose a joint network and dirty-paper coding (JNDPC) method that provides potentialities to overcome the shortcomings of JNSC and JPNC schemes. Using JNDPC as basic elements, a successive network coding (SNC) scheme is proposed. In SNC scheme, we have the relay presubstract the interference to the weaker user of each pair by using JNDPC method. The stronger user of each pair, on the other hand, performs successive decoding to cancel the interference. It is shown that the JNDPC method outperforms the JNSC scheme. Generally the SNC scheme is not superior to the JPNC scheme. We then use JNDPC to construct an improved JPNC scheme. For the MA stage, we present a full decode multiple access and a functional decode multiple access schemes, where the relay decodes respectively the full packets and a function of packets of each pair. The achievable rate regions of all proposed BC and MA schemes are derived.

The remainder of the paper is organized as follows. In section II, the system model is presented. The JNDPC method along with SNC and improved JPNC schemes are presented in section III. Section IV presents the full decode and the functional decode multiple access schemes. Some numerical results are given in section V. Section VI concludes this paper.

## II. System Model

Consider a relay network as depicted in Fig.1, which consists of $K$ pairs of user nodes $(A_i, B_i),\ i = 1, \ldots, K$ and a relay node $R$. Each user $A_i$ and its partner $B_i$ wish to exchange their packets $W_{A_i}$ and $W_{B_i}$ respectively, with the help of the relay node. There is no direct link any two user nodes, hence the packets can only be sent through a two hop transmission, namely, an uplink transmission and a downlink transmission. In the uplink transmission, the users transmit signals to the relay. In the downlink transmission, the relay broadcast a signal to all the users. Full-duplex communication is assumed so that the MA and BC transmissions do not interfere with each other.

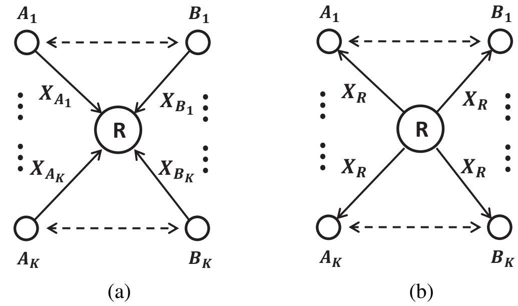


Fig. 1: Relay Networks with Pairwise Information Exchange. (a) Uplink Transmission. (b) Downlink Transmission.

### A. Channel Model

The uplink and downlink transmissions can be characterized by Gaussian MAC channel and Gaussian BC channel respectively. Let the channel gain between node $L$ and another node $V \in \{R, A_1, B_1, \ldots, A_K, B_K\} \backslash L$ be represented by $h_{LV}$. The signal transmitted by node $L = R, A_1, B_1, \ldots, A_K, B_K$ is a length-n sequence of symbols $X_L = (X_L(1), \ldots, X_L(n))$ such that $\frac{1}{N}\sum_{t=1}^{n} X_L(t) \leq P_L$. Here for $L = A_1, B_1, \ldots, A_K, B_K$, $P_L = P|h_{LR}|^2$ is the equivalent constraint power. The received symbols $Y_L$ at each node $L$ can be given by

$$\begin{aligned} Y_R(t) &= \sum_{i=1}^{K}(X_{A_i}(t) + X_{B_i}(t)) + Z_R(t), \\ Y_L(t) &= X_R(t) + Z_L(t), \qquad L = A_1, B_1, \ldots, A_K, B_K. \end{aligned} \tag{1}$$

The equivalent noise components $Z_L(t)$ are white Gaussian variables with zero mean and variance $\sigma_L^2$, where $\sigma_L^2 = \sigma^2/h_{RL}^2$ for $L = A_1, B_1, \ldots, A_K, B_K$. Without loss of generality, it is assumed that $\sigma_{A_i}^2 \leq \sigma_{B_i}^2,\ i = 1, \ldots, K$ and $\sigma_{A_1}^2 \geq \ldots \geq \sigma_{A_K}^2$

Let $|W_L|,\ L = A_1, B_1, \ldots, A_K, B_K$ denotes the length of the packet $W_L$ in bits. Then the data rate of $W_L$ is given by

$$R_L = \frac{|W_L|}{n}. \tag{2}$$

The main problem of this paper is to study the achievable data rate tuples $(R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K})$.

### B. Decode-and-Forward Schemes

We consider two-stage DF schemes, which consist of a MA stage and a BC stage, respectively for the uplink and downlink transmissions. In the BC stage, each user knows the packet it sends as side information. Hence it's possible for employment of network coding, where functions of packets of each pair $f_i(W_{A_i}, W_{B_i}),\ i = 1, \ldots, K$ are broadcast and each user decodes its desired packet based on its side information. In the MA stage, since the relay broadcasts functions of packets $f_i(W_{A_i}, W_{B_i})$ in the BC stage, it only needs to decode functions $f_i(W_{A_i}, W_{B_i})$. Note that the uplink and downlink transmissions are independent. The coding procedure and the achievable rate region can be treated separately once the functions $f_i(W_{A_i}, W_{B_i})$ are determined.

For pairwise information exchange, the functions $f_i(W_{A_i}, W_{B_i})$ are usually taken to be the bitwise XOR of packets, where zeros are added for unequal packet length. Throughout this paper, we refer to such bitwise XOR as modulo sum. And we denote $W_L \oplus W_V$ as the modulo sum of packets $W_L$ and $W_V$.

## III. BC Stage

In this section we propose a joint network and dirty-paper coding (JNDPC) method for the BC stage. Based on the JNDPC method, we construct a successive network coding scheme and an improved JPNC scheme achieve larger rate regions than those of JNSC and JNDPC respectively.

### A. Joint Network and Dirty-Paper Coding Method

The underlying coding technique of the dirty-paper coding [11] is the Gelfand-Pinsker coding scheme, which is proposed for point to point channels with state. We shall combine the Gelfand-Pinsker coding with network coding to implement the joint network and dirty-paper coding method.

Consider a two-receiver broadcast channel $\boldsymbol{P}_{Y_1,Y_2|X,S}$ with input $X$, channel state $S$ and outputs $Y_1, Y_2$. The state $S$ is noncausally available at the sender, who wants to send packets $(W_1, W_2)$ to receiver 1 and 2 respectively. Each receiver knows a priori the packet intended for the other.

**Theorem 1.** The rate pair $(R_1, R_2)$ is achievable for the above broadcast channel $\boldsymbol{P}_{Y_1,Y_2|X,S}$ if

$$\begin{aligned} R_1 &\leq I(U; Y_1) - I(U; S), \\ R_2 &\leq I(U; Y_2) - I(U; S), \end{aligned} \tag{3}$$

for some conditional pmf $p(u|s)$ and function $f(u, s)$. Here $U$ is the auxiliary random variable.

*Proof:* The proof combines index coding and Gelfand-Pinsker coding (see [12]). Fix the conditional pmf $p(u|s)$ and the function $f(u|s)$. Let $R = \max(R_1, R_2)$ be the data rate of packet $W_1 \oplus W_2$. Denote $\widetilde{W}_i$ the packet receiver $i,\ i = 1, 2$ knows a priori.

**Codebook Generation:** Generate $2^{nR}$ bins, each consists of $2^{n\widetilde{R}}$ sequences $U^n$, randomly and independently generated according to $p(u)$.

**Encoding:** To send packets $W_1$ and $W_2$ under the interference $S$, the sender chooses in bin $W = W_1 \oplus W_2$ a sequence $U^n$ such that $U^n$ is jointly $\epsilon$-typical with the interference sequence $S^n$. Then it sends the signal $X^n = f(U^n, S^n)$.

**Decoding:** Based on the received signal $Y_i^n$, receiver $i$ looks in bins $W' = W_1 \oplus W_2$, where $\widetilde{W}_i$ is fixed, for the unique sequence $U^n$ such that $U^n$ is jointly $\epsilon$-typical with $Y_i^n$. If there are none or more than such sequence $U^n$, receiver $i$ declares an error. Otherwise it declares that $W' \oplus W_i$ is sent.

**Analysis of the probability of error:** An error is attributed to the following events.

1) There is no sequence $U^n$ in bin $W$ that is jointly $\epsilon$-typical with $S^n$. Since a sequence $U^n$ satisfies $(U^n, S^n) \in A_{\epsilon'}^{(n)}$ with probability not less than $2^{n(I(U;S)+\delta)}$ [12], the probability of this event can be upper bounded by $(1 - 2^{n(I(U;S)+\delta)})^{\widetilde{R}} \leq exp(-2^{n(\widetilde{R}-I(U;S)-\delta)})$.
2) The chosen sequence $U^n$ in the encoding process is not $\epsilon$-typical with $Y_i^n$. By the weak law of large numbers, the probability tends to zero as $n$ goes to infinity.
3) In bins $W' = W_1 \oplus W_2$, where $W_i$ is fixed, there are more than one sequences $U^n$ that is jointly $\epsilon$-typical with $Y_i^n$. The probability that a sequence $U^n$ is jointly $\epsilon$-typical with variable $Y_i^n$ is not greater than $2^{n(I(U;Y_i)-\epsilon')}$ [12] Note that there are $2^{n(R_i+\widetilde{R})}$ sequences $U^n$ with the same $\widetilde{W}_i$ fixed. The probability of this event can be upper bounded by $2^{n(R_1+\widetilde{R}-I(U;Y_1)+\epsilon_1')} + 2^{n(R_2+\widetilde{R}-I(U;Y_2)+\epsilon_2')}$.

Therefore, $P(E_2)$ and $P(E_3)$ tend to zero as $n$ goes to zero, as long as $\widetilde{R} > I(U;S)+\delta$, $R_i < I(U;Y_i)-I(U;S)-\epsilon_i'-\delta$, $i = 1, 2$. ■

**Remark 1.** Note that both users are able to decode the auxiliary random variable sequence $U^n$. This fact plays a crucial role in successive network coding.

Setting $U$ in (3) to be the dirty-paper coding auxiliary random variable, we have the following corollaries.

**Corollary 1.** Suppose the broadcast channel $\boldsymbol{P}_{Y_1,Y_2|X,S}$ is given by

$$Y_i = X + S + Z_i, \ i = 1, 2 \tag{4}$$

where $X$ has average power $P$. The noise components $Z_i \sim \mathcal{N}(0, \sigma_i^2)$ and the interference $S \sim \mathcal{N}(0, Q)$ are independent. The rate pair $(R_1, R_2)$ is achievable if

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\Big(\frac{P}{\sigma_1^2}\Big), \\ R_2 &\leq \frac{1}{2}\log\Big(\frac{(P+Q+\sigma_2^2)(P+\sigma_1^2)^2}{Q\sigma_1^4+\sigma_2^2[(P+\sigma_1^2)^2+PQ]}\Big). \end{aligned} \tag{5}$$

*Proof:* Substituting $U \sim \mathcal{N}(\alpha S, P)$, $X = U - \alpha S$, and $\alpha = \frac{P}{P+\sigma_1^2}$, into (3), we get the desired region (5). ■

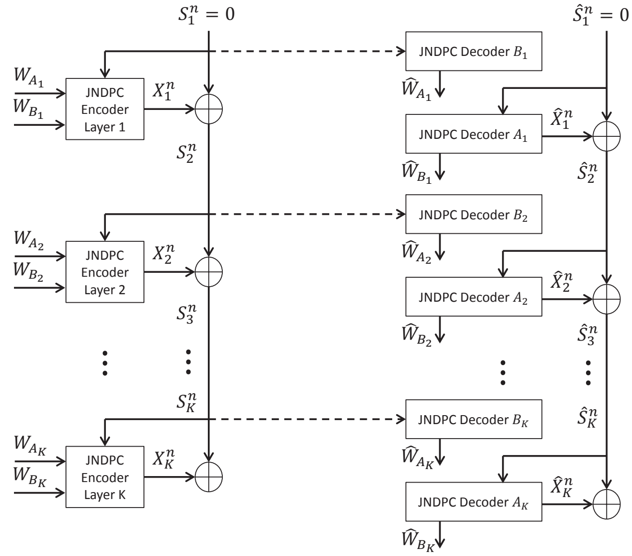


Fig. 2: Coding and Decoding Procedures of SNC Scheme

Further assuming that the interference $S$ is available also at one of the receivers, say, receiver 1, we have the following conclusion,

**Corollary 2.** The achievable rate region of channel (4), where receiver 1 knows the interference $S$ noncausally, contains the rate tuple $(R_1, R_2)$ satisfying

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\Big(\frac{P}{\sigma_2^2}\Big), \\ R_2 &\leq \frac{1}{2}\log\Big(\frac{P}{\sigma_1^2}\Big). \end{aligned} \tag{6}$$

*Proof:* Since receiver 1 knows the interference $S$, the proof and conclusion still hold if $S$ is incorporated into $Y_1 = (Y_1, S)$ [13], which yields $R_1 = I(U; Y_1, S) - I(U; S) = I(U; Y_1|S)$ in (3). The rest step is to set $U \sim \mathcal{N}(\alpha S, P)$, $X = U - \alpha S$, and $\alpha = \frac{P}{P+\sigma_2^2}$ ■

We shall refer to the coding schemes that achieve the rate region (5) and (6) as joint network and dirty-paper coding.

### *B. Successive Network Coding*

In the JNSC scheme, the modulo sums of packets are superimposed for broadcast. When employing successive decoding, each user cancel the interference from packets whose destined pair of users both have relatively worse channel conditions. The interference cancelation performance of each user is limited by its equivalent noise power.

To overcome such shortcoming, we propose a successive network coding (SNC) scheme, where the relay and the users cooperate to cancel interference. Rather than transmitting superimposed signals of each pair, the relay aligns the signals in a layered structure, where it uses JNDPC to presubstract the interference to the weaker user of the pair in each layer. Besides, successive decoding is employed at the stronger user of each pair in order to further cancel the interference. The coding and decoding procedures of SNC scheme are shown in Fig.2. The following describes the successive network coding scheme in detail.

**Encoding at Relay:** Generate the $K$-layer signal $X_R$ as

$$X_R^n = X_1^n(W_{A_1} \oplus W_{B_1}) + \ldots + X_K^n(W_{A_K} \oplus W_{B_K}). \quad (7)$$

In the $k$th layer, consider $S_k^n = \sum_{m=1}^{k-1} X_m^n(W_{A_m} \oplus W_{B_m})$ as interference, and then generate a codebook $\boldsymbol{D}(k)$ of sequences $U_k^n$, where $\alpha_k = \frac{P_k}{\sigma_{B_k}^2 + P_k}$ and $U_k \sim \mathcal{N}(\alpha_k S, P_k)$, by employing JNDPC method. Finally chose a sequence $U_k^n$ and generate $X_k^n(W_{A_k} \oplus W_{B_k}) = U_k^n - \alpha_k S_k^n$.

**Successive decoding at user $A_i$:** Decode the auxiliary random variable sequence $U_k^n$ from the first to the $i$th layer sequentially. In the $k$th ($k \leq i$) layer, consider $\hat{S}_k^n$ ($hatS_1 = 0$) as interference and treat $\sum_{j=k+1}^{K} X_j^n(W_{A_j} \oplus W_{B_j})$ as additive noise. Then look in codebook $\mathcal{D}(k)$ for the unique sequence $\hat{U}_k^n$ that is jointly typical with $Y_{A_i}^n, \hat{S}_k^n$ and calculate the sequence $\hat{X}_k^n = \hat{U}_k^n - \alpha_k \hat{S}_k^n$. Then renew the interference $\hat{S}_{k+1}^n = \hat{S}_k^n + \hat{X}_k^n$. Finally determine the bin $W$ containing sequence $\hat{U}_i^n$ and decode the packet $\hat{W}_{B_i} = W \oplus W_{A_i}$

**Direct decoding at user $B_i$:** Treat $\sum_{j=i+1}^{K} X_j^n(W_{A_j} \oplus W_{B_j})$ as additive noise. Then look in codebook $\mathcal{D}(i)$ for the unique sequence $\hat{U}_i^n$ that is jointly typical with $Y_{B_i}^n$. Then determine the bin $W$ containing $U_i^n$ and decode the packet $\hat{W}_{A_i} = W \oplus W_{B_i}$.

The correctness of successive decoding at user $A_i$ relies on the fact that both users can decode the auxiliary random variable sequence $U^n$ in the JNDPC method (see Remark 1). The data rate region achieved by successive network coding scheme is established in the following theorem.

**Theorem 2.** For the BC stage, the achievable rate region achieved by the SNC scheme is given by

$$\begin{aligned} \mathcal{C}^{SNC} = \Big\{ &(R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K}) : \\ &R_{A_i} \leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{B_i}^2 + \sum_{m=k+1}^{K} P_m}\Big), \\ &R_{B_i} \leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{A_i}^2 + \sum_{m=k+1}^{K} P_m}\Big), \\ &\sum_{m=1}^{K} P_m = P_R \Big\}. \end{aligned} \quad (8)$$

*Proof:* Fix the power tuple $(P_1, \ldots, P_K)$ and the data rate tuple $(R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K}) \in \mathcal{C}^{JNDPC}(P_1, \ldots, P_K)$.

First, since user $A_1$ knows the interference $S_1 = 0$, it follows from Corollary 1 that the users $A_1$ and $B_1$ are able to decode the sequence $U_1^n$ and the packets $W_{B_1}$ and $W_{A_1}$ respectively.

Next, note that $|W_{B_k}| = |W_{A_k} \oplus W_{B_k}|$. According to the generation of codebook of sequences $U^n$ and the decoding procedure in Lemma 1, knowing $W_{A_k}$ a priori will not help $A_k$ reduce the size of the codebook it looks up for the chosen sequence $U_k^n$. Then since $A_i$ is stronger than $A_k,\ k < i$, it can decode the sequence $U_k^n,\ k < i$ and thus the interference $\hat{S}_k^n = S_k^n$ correctly if $A_k$ is able to decode $U_k^n$. On the other hand, given that $A_i$ knows the interference $S_k^n$, it follows from Corollary 1 and Remark 1 that users $(A_i, B_i)$ decode the sequence $U_i^n$ and the packets $W_{B_i}$ and $W_{A_i}$ correctly since

$$\begin{aligned} R_{A_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{B_i}^2 + \sum_{m=i+1} KP_m}\Big), \\ R_{B_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{A_i}^2 + \sum_{m=i+1} KP_m}\Big). \end{aligned} \quad (9)$$

It then follows by mathematical induction that each pair of users $(A_i, B_i)$ decode $U_i^n$ and their desired packets successfully. Thus the theorem is proved. ■

**Remark 2.** The rate region $\mathcal{C}^{SNC}$ is not convex on $(P_1, \ldots, P_K)$ in general. By employing time sharing among different power allocation strategies, the convex hull of $\mathcal{C}^{SNC}$ can be achieved.

We now show that the rate region $\mathcal{C}^{SNC}$ is larger than that of JNSC scheme. The transmit signal in the JNSC scheme is given by

$$X_R = X_1(W_{A_1} \oplus W_{B_1}) + \ldots + X_K(W_{A_K} \oplus W_{B_K}), \quad (10)$$

where $X_i(W_{A_i} \oplus W_{B_i}),\ i = 1, \ldots, K$ are independent Gaussian variables. By employing successive decoding, the rate tuple $\boldsymbol{R} = (R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K})$ is achievable if

$$\begin{aligned} R_{A_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{B_i}^2 + \sum_{\sigma_{A_m}^2 < \sigma_{B_i}^2} P_m}\Big), \\ R_{B_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{A_i}^2 + \sum_{m=i+1}^{K} P_m}\Big), \end{aligned} \quad (11)$$

for some power tuple $(P_1, \ldots, P_K)$ such that $\sum_{m=1}^{K} P_m = P_R$. For any $\boldsymbol{R}$ satisfying (11), we have

$$\begin{aligned} R_{A_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{B_i}^2 + \sum_{\sigma_{A_m}^2 < \sigma_{A_i}^2} P_m}\Big) \\ &= \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{B_i}^2 + \sum_{m=i+1}^{K} P_m}\Big), \\ R_{B_i} &\leq \frac{1}{2}\log\Big(1 + \frac{P_i}{\sigma_{A_i}^2 + \sum_{m=i+1}^{K} P_m}\Big), \end{aligned} \quad (12)$$

which implies that $\boldsymbol{R}$ is an interior point of $\mathcal{C}^{SNC}$. Hence, the rate region $\mathcal{C}^{SNC}$ is larger than the rate region characterized by (11).

It is worth to note that the pure time sharing scheme, where the modulo sums are sent in different time slots, can be regarded as time sharing among SNC schemes under different power allocation strategies. Hence the achievable rate region of pure time sharing scheme is contained in the convex hull of $\mathcal{C}^{SNC}$.

### C. Improved JPNC Method

In the JPNC scheme, network coding and dirty-paper coding are separated. The packets $W_{B_i}$ are splited into packets $W_{B_i}^1$ and $W_{B_i}^2$ such that $|W_{B_i}^1| = |W_{A_i}^2|$. The packets $W_{A_i} \oplus W_{B_i}^1$

and $W_{B_i}^2$ are successively encoded using dirty-paper coding. The transmitted signal is given by

$$\begin{aligned} X_R = X_{\pi(A_1)}(W_{A_1} \oplus W_{B_1}^1) + X_{\pi(B_1)}(W_{B_1}^2) + \dots \\ + X_{\pi(A_K)}(W_{A_K} \oplus W_{B_K}^1) + X_{\pi(B_K)}(W_{B_K}^2), \end{aligned} \tag{13}$$

where $\pi$ is the coding order. At the users, self information substraction is further used for interference cancelation. It is shown in [10] that any rate tuple $\boldsymbol{R} = (R_{A_i}, R_{B_i}^1, R_{B_i}^2 : i = 1, \dots, K)$ is achievable in JPNC scheme if

$$\begin{aligned} & R_{A_i} = R_{B_i}^1 \le \min\{\frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{A_i}'^2}\Big), \\ & \frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{B_i}'^2}\Big)\} \\ & R_{B_i}^2 \le \frac{1}{2}\log\Big(1 + \frac{P_{\pi(B_i)}}{\sigma_{A_i}^2 + \delta_{\pi(B_i)}^2}\Big), \end{aligned} \tag{14}$$

for some encoding order $\pi$. Here $\delta_{\pi(L)}^2 = \sum_{\pi(M)>\pi(L)} P_{\pi(M)}$, $L = A_1, B_1, \dots, A_K, B_K$ is the interference from undecoded signals when decoding $W_{A_1} \oplus W_{B_1}^1$. In JPNC scheme, both the data rate $R_{A_i}$ and $R_{B_i}^1$ are constrained by the minimum signal interference plus noise ratio of users $A_i$ and $B_i$.

Let us denote $\sigma_{A_i}'^2 = \sigma_{A_i}^2 + \delta_{\pi(A_i)}^2$ and $\sigma_{B_i}'^2 = \sigma_{B_i}^2 + \delta_{\pi(A_i)}^2 - I_{\{\pi(A_i)<\pi(B_i)\}} P_{\pi(Bi)}$ as the equivalent noise power of user $A_i$ and $B_i$ respectively when decoding $W_{A_1} \oplus W_{B_1}^1$. We propose an improved JPNC coding scheme by using JNDPC method as follows.

1) **Step 1:** Split packet $W_{B_i}$ into packets $W_{B_i}^1$ and $W_{B_i}^2$. Then generate a new packet $W_{A_i} \oplus W_{B_i}^1$.
2) **Step 2:** Generate $X_{\pi(B_i)}(W_{B_i}^2)$ by using dirty-paper coding. Generate $X_{\pi(A_i)}(W_{A_i} \oplus W_{B_i}^1)$ by using JNDPC, where $S = \sum_{\pi(L)<\pi(A_i)} X_{\pi(L)}$, $U \sim \mathcal{N}(\alpha_i S, P)$, and $\alpha_i = \frac{P}{\min\{\sigma_{A_i}'^2, \sigma_{B_i}'^2\}+P}$.
3) **Step 3:** Transmit $X_R = \sum_L X_{\pi(L)}$.

Applying the result in Corollary 1, we obtain the similar result as in (14).

**Theorem 3.** The rate region achieved by the improved JPNC scheme is given by

$$\begin{aligned} & \mathcal{C}^{IMJPNC} = \Big\{(R_{A_1}, R_{A_2}, \dots, R_{A_K}, R_{B_K}) : \\ & R_{A_i} \le I_{\{\sigma_{A_i}'^2 > \sigma_{B_i}'^2\}} \frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{A_i}'^2}\Big) \\ & + I_{\{\sigma_{A_i}'^2 \le \sigma_{B_i}'^2\}} \frac{1}{2}\log\Big(\frac{(P_i + Q_i + \sigma_{A_i}'^2)(P_i + \sigma_{B_i}'^2)^2}{Q_i \sigma_{B_i}'^4 + \sigma_{A_i}'^2[(P_i + \sigma_{B_i}'^2)^2 + P_i Q_i]}\Big), \\ & R_{B_i}^1 \le I_{\{\sigma_{A_i}'^2 \le \sigma_{B_i}'^2\}} \frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{B_i}'^2}\Big) \\ & + I_{\{\sigma_{A_i}'^2 > \sigma_{B_i}'^2\}} \frac{1}{2}\log\Big(\frac{(P_i + Q_i + \sigma_{B_i}'^2)(P_i + \sigma_{A_i}'^2)^2}{Q_i \sigma_{A_i}'^4 + \sigma_{B_i}'^2[(P_i + \sigma_{A_i}'^2)^2 + P_i Q_i]}\Big), \\ & R_{B_i}^2 \le \frac{1}{2}\log\Big(1 + \frac{P_{\pi(B_i)}}{\sigma_{A_i}^2 + \delta_{\pi(B_i)}^2}\Big) \\ & \sum_{i=1}^{K}(P_{A_i}, P_{B_i}) = P_R\Big\}, \end{aligned} \tag{15}$$

where $P_i = P_{\pi(A_i)},\ Q_i = Q_{\pi(A_i)}$.

**Remark 3.** In general, neither of the rate regions $\mathcal{C}^{IMJPNC}$ and $\mathcal{C}^{SNC}$ contains the other. Hence time sharing among these two schemes can be used to enlarge the achievable rate region.

It is easy to verify that

$$R_{A_i}, R_{B_i}^1 \ge \min\{\frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{A_i}'^2}\Big), \frac{1}{2}\log\Big(1 + \frac{P_{\pi(A_i)}}{\sigma_{B_i}'^2}\Big)\}. \tag{16}$$

Hence, any rate tuple $\boldsymbol{R}$ that satisfies (14) must satisfy (15). And we conclude that the rate region characterized by (15) is larger than that characterized by (14).

## IV. MA Stage

In this section we present full decode and functional decode multiple access (MA) schemes for the MA stage. The rate regions achieved by these two schemes are characterized.

In successive network coding, the relay only needs to know the modulo sums of packets from each pair. In the improved JPNC scheme, all the packets are required to be known at the relay. Hence we present a full decode MA scheme and a functional decode MA scheme for the MA stage, which decode the full packets $W_L, L = A_1, B_1, \dots, A_K, B_K$ and the modulo sums $W_{A_i} \oplus W_{B_i},\ i = 1, \dots, K$ respectively. Both schemes are suitable for broadcasting with successive network coding. And only full decode MA scheme can be adopted when employing the improved JPNC scheme for the BC stage.

### A. Full Decode Multiple Access

To completely decode all packets $W_L$, $L = A_1, B_1, \dots, A_K, B_K$, the relay employs successive decoding.

The capacity region achieved by this scheme is given by

$$\begin{aligned}\mathcal{C}^{Full} = \Big\{ &(R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K}) : \\ &\sum_{L \in \mathcal{S}} R_L \leq \frac{1}{2} \log \Big( 1 + \frac{\sum_{L \in \mathcal{S}} P_L}{\sigma_R^2} \Big), \\ &\forall \mathcal{S} \subset \{A_1, B_1, \ldots, A_K, B_K\} \Big\}.\end{aligned} \tag{17}$$

*B. Functional Decode Multiple Access*

It has been shown in [4] that when $K = 1$, the relay can successfully decode the modulo sum $W_{A_1} \oplus W_{B_1}$ if

$$\begin{aligned} R_{A_1} &\leq \frac{1}{2} \log \Big( \frac{P_{B_1}}{P_{A_1} + P_{B_1}} + \frac{P_{A_1}}{\sigma_R^2} \Big) \Big\}, \\ R_{B_1} &\leq \frac{1}{2} \log \Big( \frac{P_{B_1}}{P_{A_1} + P_{B_1}} + \frac{P_{B_1}}{\sigma_R^2} \Big) \Big\}. \end{aligned} \tag{18}$$

The rate pair $(R_{A_1}, R_{B_1})$ can be achieved by lattice coding. In cases when $K > 1$, each pair of users perform lattice coding and the relay decodes the lattice codewords of each pair by employing successive decoding. Besides, time sharing is employed among different decoding orders $\pi$. The achievable rate region in this scheme is given by

$$\mathcal{C}^{Functional} = Conv(\bigcup_{\pi} \mathcal{C}_\pi), \tag{19}$$

where

$$\begin{aligned}\mathcal{C}_\pi = \Big\{ &(R_{A_1}, R_{B_1}, \ldots, R_{A_K}, R_{B_K}) : \\ &R_{A_{\pi(i)}} \leq \frac{1}{2} \log \Big( \frac{P_{A_{\pi(i)}}}{P_{A_{\pi(i)}} + P_{B_{\pi(i)}}} \\ &+ \frac{P_{A_{\pi(i)}}}{\sigma_R^2 + \sum_{\pi(m) > \pi(i)}^{K} (P_{A_{\pi(m)}} + P_{B_{\pi(m)}})} \Big), \\ &R_{B_{\pi(i)}} \leq \frac{1}{2} \log \Big( \frac{P_{B_{\pi(i)}}}{P_{A_{\pi(i)}} + P_{B_{\pi(i)}}} \\ &+ \frac{P_{B_{\pi(i)}}}{\sigma_R^2 + \sum_{\pi(m) > \pi(i)}^{K} (P_{A_{\pi(m)}} + P_{B_{\pi(m)}})} \Big) \Big\}.\end{aligned} \tag{20}$$

Note that the rate region $\mathcal{C}^{Functional}$ is a polyhedral.

## V. Simulation

In this section, we provide numerical results to demonstrate the performance of our proposed schemes. Consider a relay network with two pair information exchange. The equivalent noise power is set by $\sigma_{A_1}^2 = 1$, $\sigma_{B_1}^2 = 10$, $\sigma_{A_2}^2 = 4$, $\sigma_{B_2}^2 = 8$, $\sigma_R^2 = 2$. The equivalent power of the users and the relay are equal, i.e., $P_{A_1} = P_{B_1} = P_{A_2} = P_{B_2} = P_R = P$.

Figure 3 plots the maximal sum rates as a function of the relay power $P$. It is shown that SNC and the improved JPNC schemes perform better than JNSC and JPNC schemes respectively. In general, the achievable rate regions of SNC and improved JPNC schemes are larger than those of JNSC and JPNC schemes respectively. schemes outperforms the other.

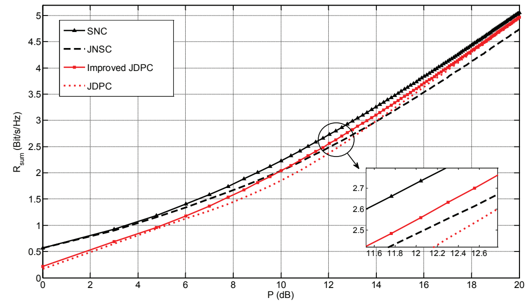


Fig. 3: Maximal Sum rate Versus Power P

## VI. Conclusion

In this paper two-stage DF schemes for relaying with pair-wise information exchange are studied. We propose a joint network and dirty-paper coding method for the BC stage, which provides potentialities to improve existing coding schemes including JNSC, JPNC schemes and their combination with time sharing. With JNDPC method, we construct a two-layer successive network coding scheme and an improved JPNC scheme that outperform JNSC and JPNC schemes respectively. A full decode multiple access scheme and a functional decode multiple access scheme are then presented to provide MA stage coding for SNC and improved JPNC schemes. The achievable rate regions of all BC and MA stage coding schemes are characterized.

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